Homeomorphisms. Surfaces

Sasha Patotski

Cornell University

ap744@cornell.edu

November 12, 2014

Definition

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A homeomorphism between two geometric figures X and Y is a **continuous bijective** map $f: X \to Y$ such that its inverse $f^{-1}: Y \to X$ is **also continuous**.

Example: Any knot is *homeomorphic* to the unknot.

Figure : But they are not isotopic!

Some examples

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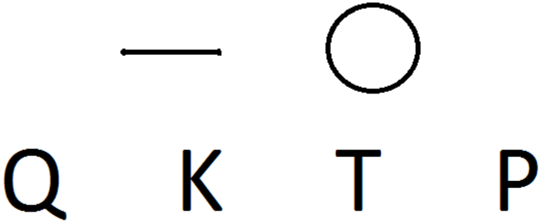
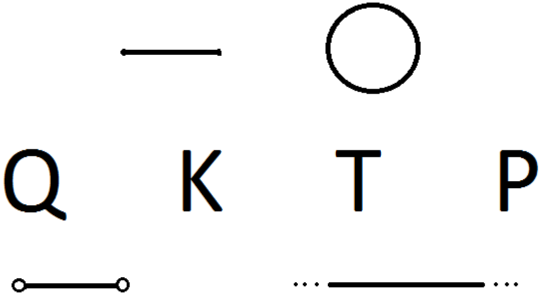


Some examples



Cylinder

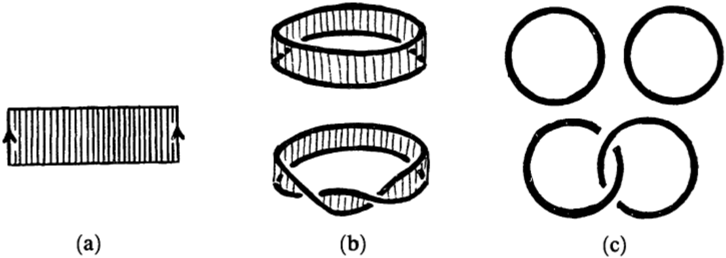
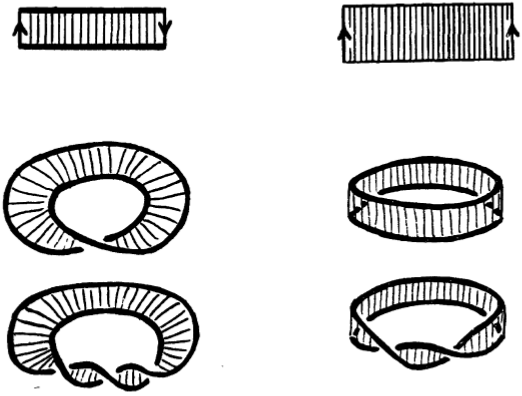
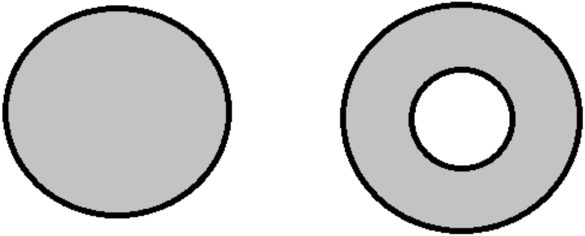


Figure : Cylinder, twisted cylinder and their boundaries

Cylinder and Möbius band



Disc and annulus



Real life application

Figure : Filled doughnut is not the same as regular doughnut!

Basic surfaces

Definition

A (closed) *surface* Σ is a geometric figure, or a subset of $\mathbb{R}^n$, such for any point $x \in \Sigma$ there exists a small neigbourhood U of x in Σ which is homeomorphic to an open disc in $\mathbb{R}^2$.

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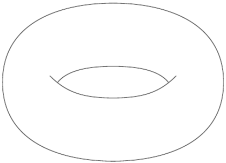
- $\mathbb{R}^2$ as well as any open subset of it is a surface.
- Sphere: any surface homeomorphic to $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$.

More surfaces

Torus:



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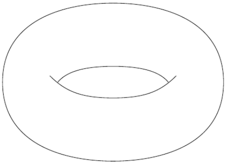

Klein bottle: